Outlier-robust estimation of uncertain-input systems with applications to nonparametric FIR and Hammerstein models

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Overview

Systems with uncertain inputs

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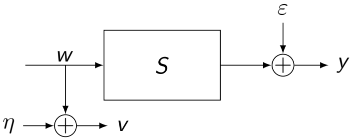

errors-in-variables

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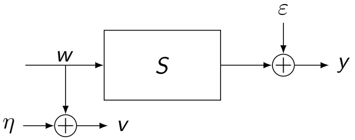

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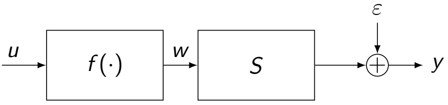
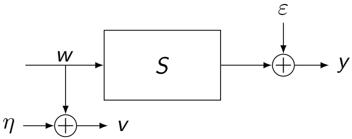


Hammerstein

Systems with uncertain inputs



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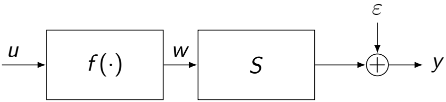

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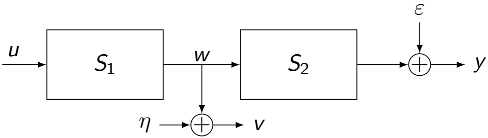

cascade

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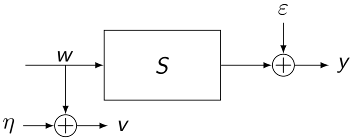

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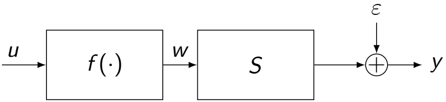

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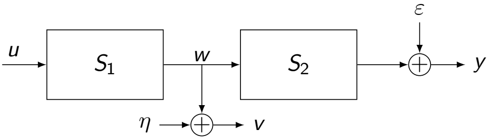

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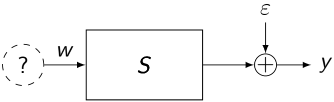
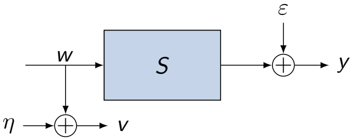


blind

Systems with uncertain inputs



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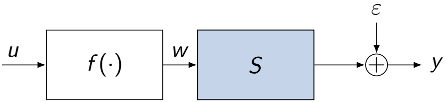

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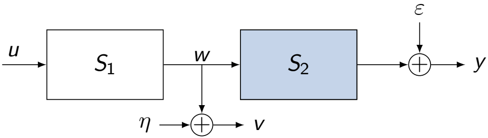

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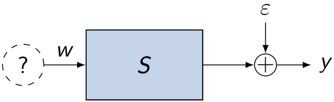
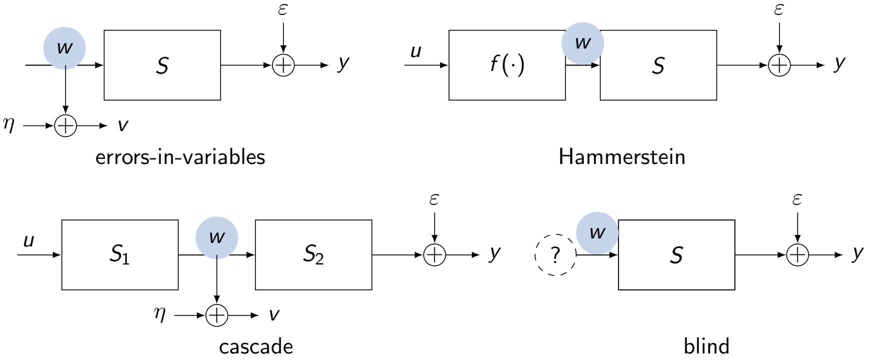


blind

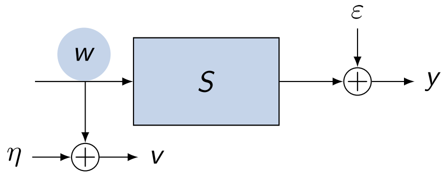
- A linear system S

Systems with uncertain inputs



- A linear system S
- An unknown input signal w

The uncertain-input system



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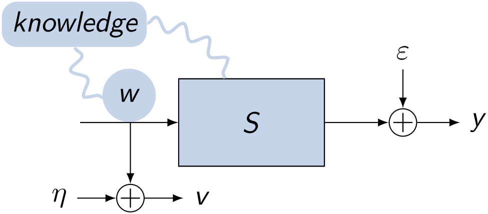

- Linear system S
- Unknown input w
- Prior knowledge

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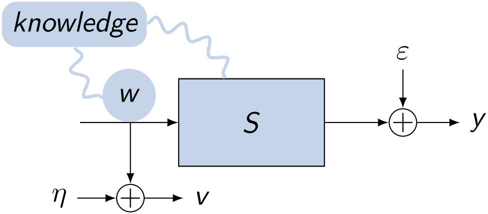

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How do we model these?

Gaussian processes

Gaussian processes

- Gaussian distribution over functions

$$f(\cdot) \sim \mathcal{N}\big(\mu(\cdot), K(\cdot, \cdot)\big)$$

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- The values of the function have a joint Gaussian distribution

$$\begin{bmatrix} f(x_1) \\ f(x_2) \\ f(x_3) \end{bmatrix} = \mathcal{N}\left(\begin{bmatrix} \mu(x_1) \\ \mu(x_2) \\ \mu(x_3) \end{bmatrix}, \begin{bmatrix} K(x_1, x_1) & K(x_1, x_2) & K(x_1, x_3) \\ K(x_2, x_1) & K(x_2, x_2) & K(x_2, x_3) \\ K(x_3, x_1) & K(x_3, x_2) & K(x_3, x_3) \end{bmatrix} \right)$$

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- Given values of the function $y = f(\tilde{x})$, we can estimate

$$\hat{f}(x) = \mathbf{E}\{f(x)|y\} = \mu(x) + K(x,\tilde{x})\left[K(\tilde{x},\tilde{x})\right]^{-1}(y - \mu(\tilde{x}))$$

Gaussian processes

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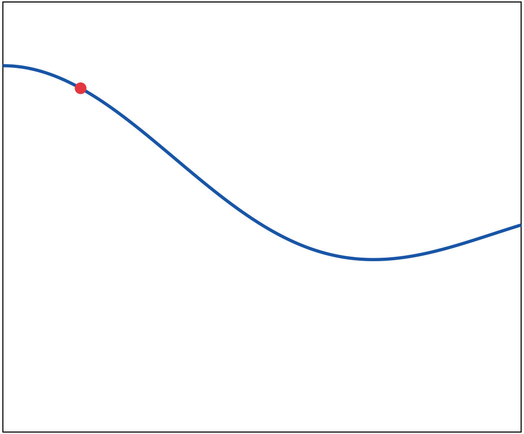

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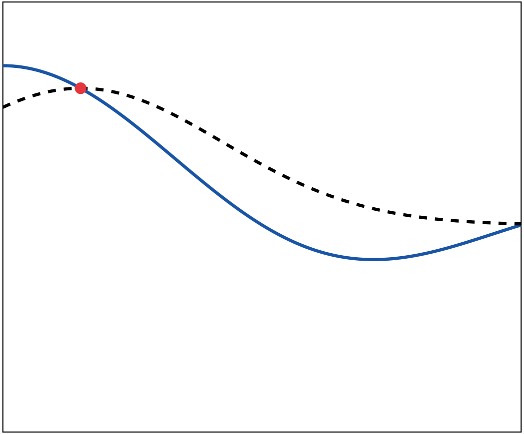

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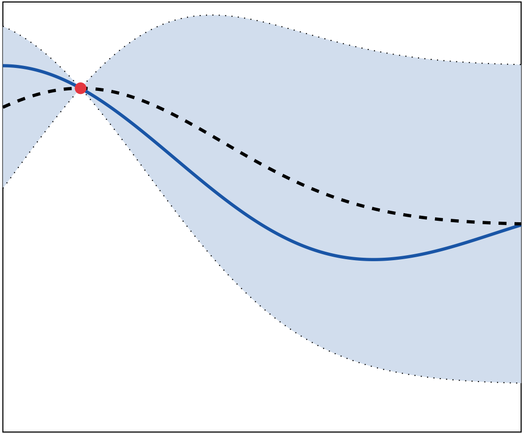

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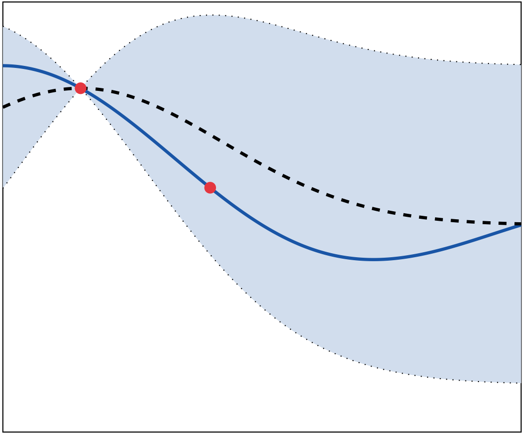

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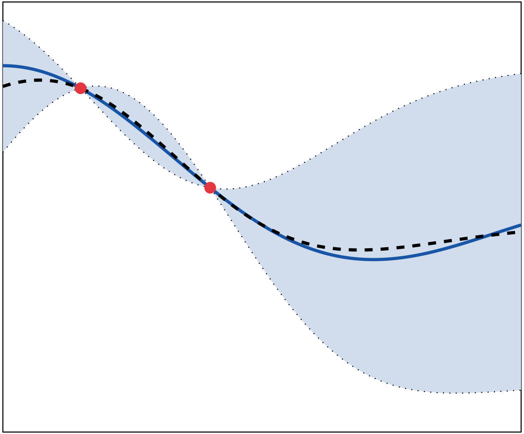

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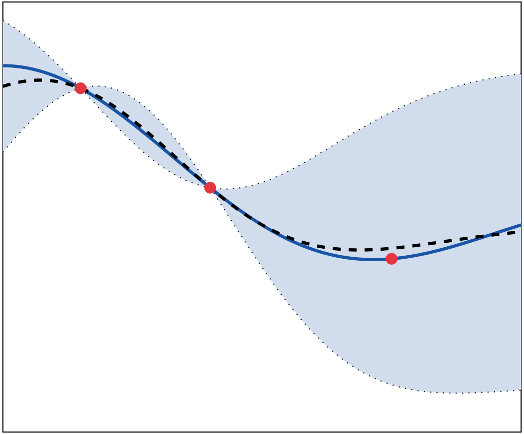

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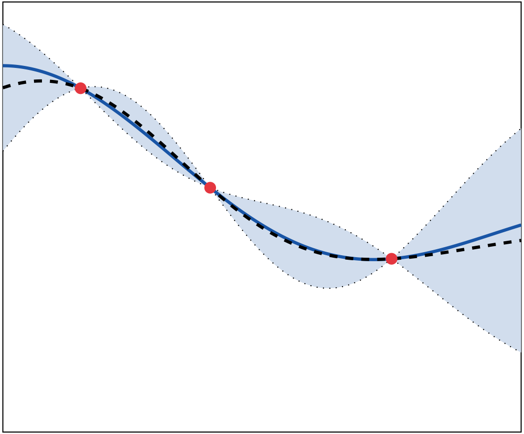
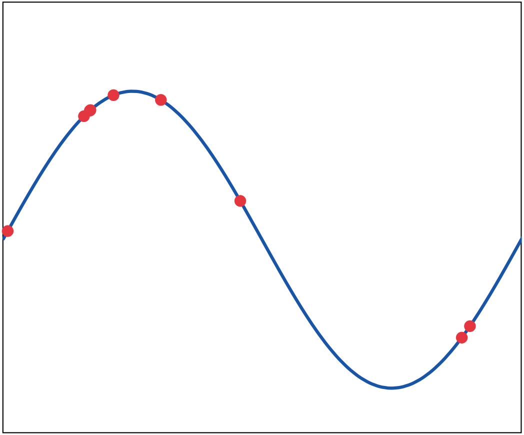


Using Gaussian processes to encode information



$$K(x_1, x_2) = e^{-\frac{1}{\theta}(x_1 - x_2)^2}$$

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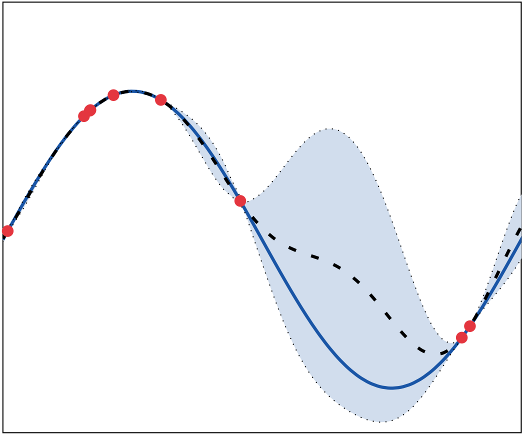

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- $\theta = 0.1$

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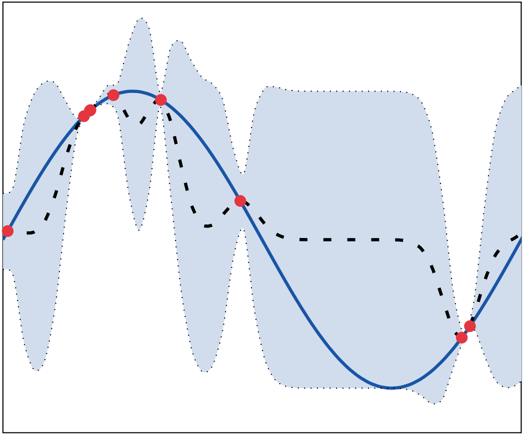

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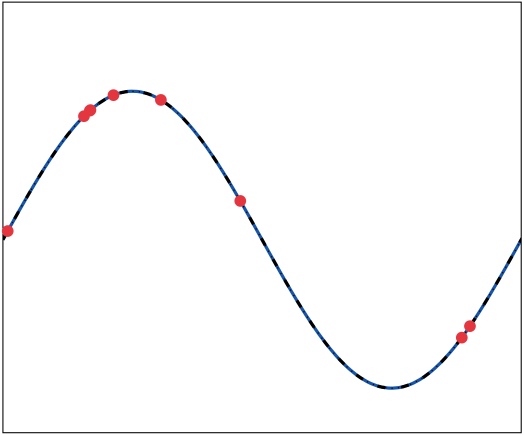
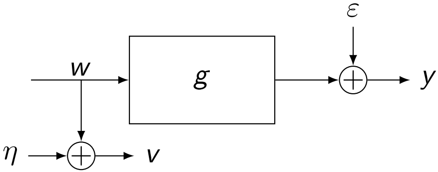


$$K(x_1, x_2) = e^{-\frac{1}{\theta}(x_1 - x_2)^2}$$

- $\theta = 0.1$
- $\theta = 0.01$
- $\theta = 1$

Modeling the uncertain input system



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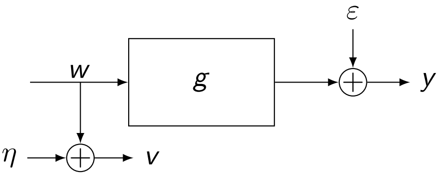

- Gaussian process prior for the impulse response

$$g \sim \mathcal{N}(\mu_g(\rho), K_g(\rho))$$

$$\left[\mu_g(\rho)\right]_i = \mathbf{E}\{g_i\} \qquad \left[K_g(\rho)\right]_{ij} = \mathbf{cov}\{g_i, g_j\}$$

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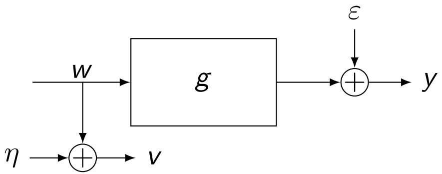

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- Gaussian process prior for the input

$$w \sim \mathcal{N}(\mu_w(\theta), K_w(\theta))$$

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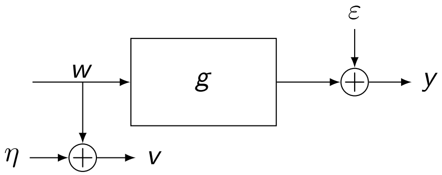
Measurement setup



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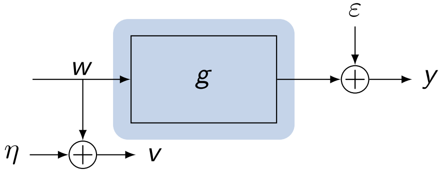

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$$y_{\text{noiseless}} = w * g$$

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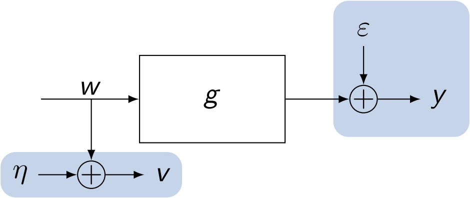


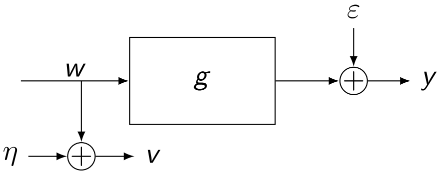
- The system is stable and LTI

$$y_{\text{noiseless}} = w * g$$

- The noises are additive, Gaussian, and white

$$v = w + \eta \qquad y = w * g + \varepsilon$$
$$\eta \sim \mathcal{N}(0, \sigma_v^2 I) \qquad \varepsilon \sim \mathcal{N}(0, \sigma_y^2 I)$$

The uncertain-input model



The uncertain-input model

$$\begin{cases} y = w * g + \varepsilon \\ v = w + \eta \\ \varepsilon \sim \mathcal{N}(0, \sigma_y^2 I) \\ \eta \sim \mathcal{N}(0, \sigma_v^2 I) \\ g \sim \mathcal{N}(\mu_g(\rho), K_g(\rho)) \\ w \sim \mathcal{N}(\mu_w(\theta), K_w(\theta)) \end{cases}$$

Inference in uncertain-input models

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$$\hat{g} = \mathbf{E}\{g|v, y\} \qquad \hat{w} = \mathbf{E}\{w|v, y\}$$

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$$\hat{g} = \mathbf{E}\{g|v,y\} \qquad \hat{w} = \mathbf{E}\{w|v,y\}$$

- We do not know the hyperparameters $\tau = \{\rho, \theta, \sigma_v^2, \sigma_y^2\}$!

$$\hat{g} = \hat{g}(\tau) \qquad \hat{w} = \hat{w}(\tau)$$

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We need to estimate them from data

Empirical Bayes

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- We choose the hyperparameters that maximize the *marginal likelihood*

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- Estimates:

$$\hat{g}(\hat{\tau}) = \mathbf{E}\{g|v, y; \hat{\tau}\} \qquad \hat{w}(\hat{\tau}) = \mathbf{E}\{w|v, y; \hat{\tau}\}$$

Is it really that simple?

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Short answer

Yes

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Yes

Long answer

Yes, but...

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- we need to calculate expected values

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- some distributions are not available in closed form

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- we need to calculate expected values $\leftarrow$ Monte Carlo
- some distributions are not available in closed form $\leftarrow$ Gibbs
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Long answer

Yes, but...

- we need to calculate expected values $\leftarrow$ Monte Carlo
- some distributions are not available in closed form $\leftarrow$ Gibbs
- we need to maximize the marginal likelihood $\leftarrow$ EM

Calculating the posterior mean

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$$\mathbf{E}\left\{g|y,v;\tau\right\}$$

$$\mathbf{E}\left\{w|y,v;\tau\right\}$$

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$$\mathbf{E}\left\{g|y,v;\tau\right\} = \int g\, p(g,w|y,v;\tau)\,\mathrm{d}w\,\mathrm{d}g$$
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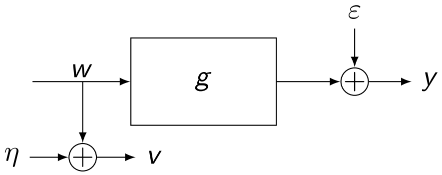

Monte Carlo integration

Monte Carlo integration

- We make a particle approximation of the distribution

$$p(g, w|y, v; \tau) \approx \frac{1}{M} \sum_{j=1}^{M} \delta(g - \bar{g}^{(j)}, w - \bar{w}^{(j)})$$

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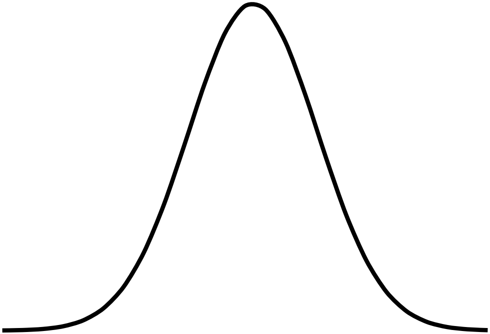

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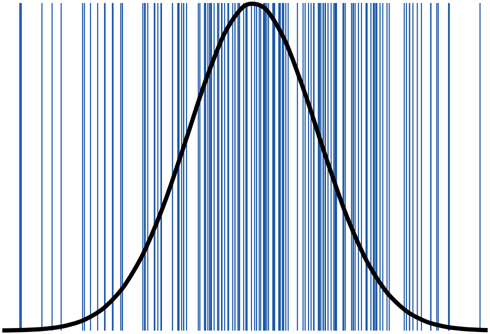

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$$p(g, w|y, v; \tau) \approx \frac{1}{M} \sum_{j=1}^{M} \delta(g - \bar{g}^{(j)}, w - \bar{w}^{(j)})$$

- We approximate the posterior mean

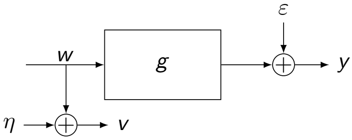
$$\begin{aligned} \mathbf{E}\{g|y, v\} &= \int g\, p(g, w|y, v; \tau)\, \mathrm{d}w\, \mathrm{d}g \\ &\approx \int g \frac{1}{M} \sum_{j=1}^{M} \delta(g - \bar{g}^{(j)}, w - \bar{w}^{(j)})\, \mathrm{d}w\, \mathrm{d}g = \frac{1}{M} \sum_{j=1}^{M} \bar{g}^{(j)} \end{aligned}$$

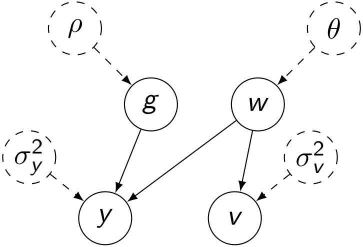
- Similarly

$$\mathbf{E}\{w|y, v\} = \int w\, p(g, w|y, v; \tau)\, \mathrm{d}g\, \mathrm{d}w \approx \frac{1}{M} \sum_{j=1}^{M} \bar{w}^{(j)}$$

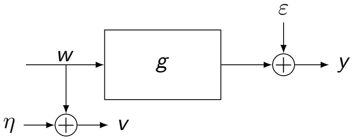
Particle approximation

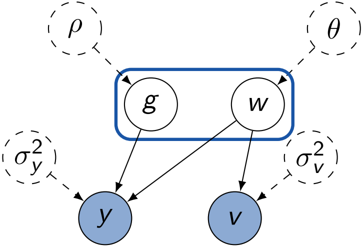
Particle approximation





Particle approximation

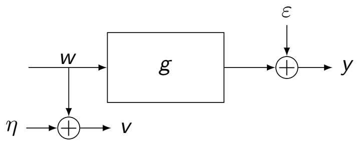


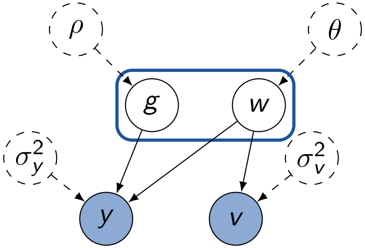


- We want samples from a joint distribution

$$p(g, w | y, v; \tau)$$

Particle approximation

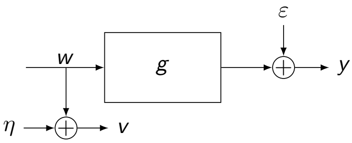


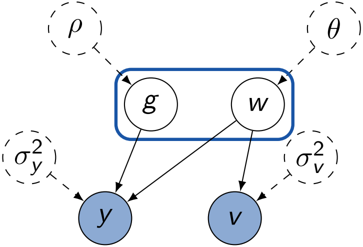


- We want samples from a joint distribution

$$p(g, w|y, v; \tau) \leftarrow \textit{difficult to evaluate!}$$

Particle approximation





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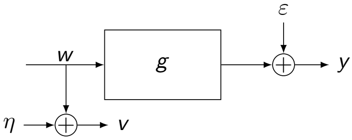
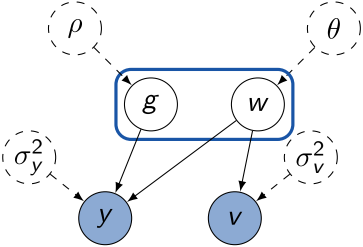
Particle approximation





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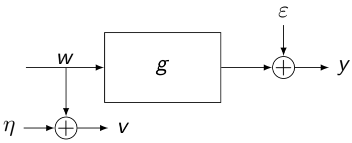

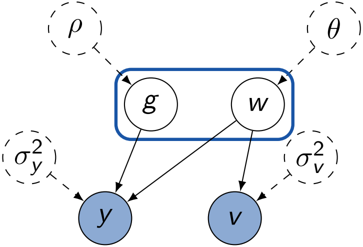

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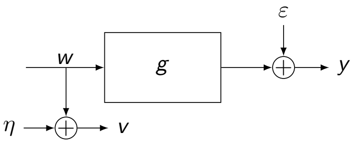

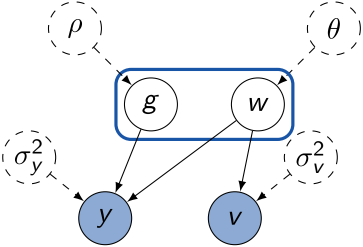

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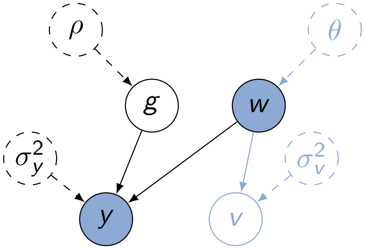
The Gibbs sampler



$$g|y, w \sim \mathcal{N}$$

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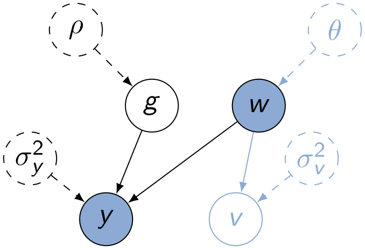

$g|y, w \sim \mathcal{N}$

$$\bar{g}^{(k+1)} \sim p(g|y, \bar{w}^{(k)}; \hat{\rho}, \hat{\sigma}_y^2),$$

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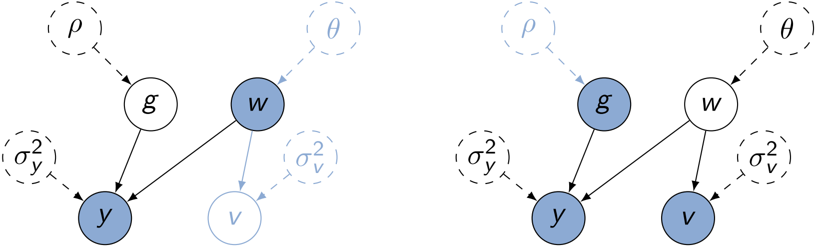

$g|y, w \sim \mathcal{N}$ $\quad$ $w|g, v, y \sim \mathcal{N}$

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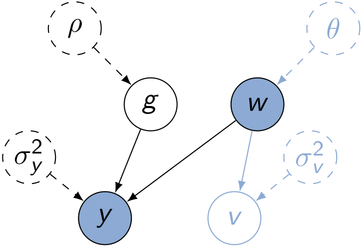

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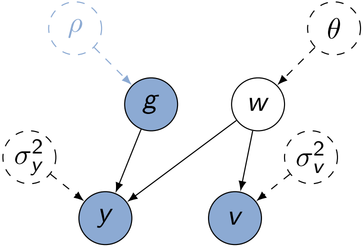

$w|g, v, y \sim \mathcal{N}$

$$\bar{g}^{(k+1)} \sim p(g|y, \bar{w}^{(k)}\,; \hat{\rho}, \hat{\sigma}_y^2),$$
$$\bar{w}^{(k+1)} \sim p(w|y, v, \bar{g}^{(k+1)}\,; \hat{\theta}, \hat{\sigma}^2)$$

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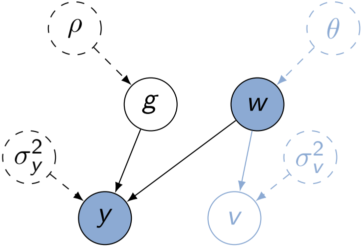

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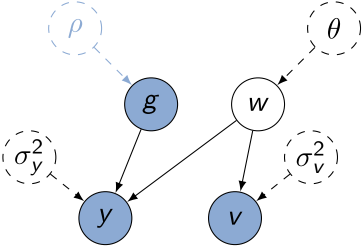
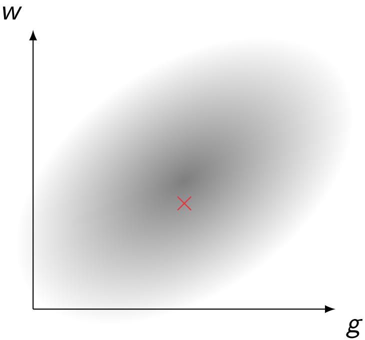


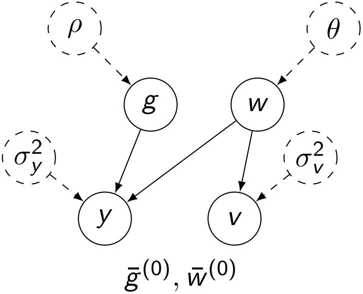
$w|g, v, y \sim \mathcal{N}$

$$\bar{g}^{(k+1)} \sim p(g|y, \bar{w}^{(k)}; \hat{\rho}, \hat{\sigma}_y^2),$$
$$\bar{w}^{(k+1)} \sim p(w|y, v, \bar{g}^{(k+1)}; \hat{\theta}, \hat{\sigma}^2)$$

$(\bar{w}^{(k)}, \bar{g}^{(k)})$ are samples from $p(g, w|y, v; \hat{\tau})$

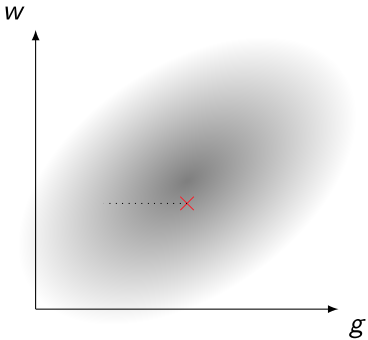
Gibbs sampling the UI model

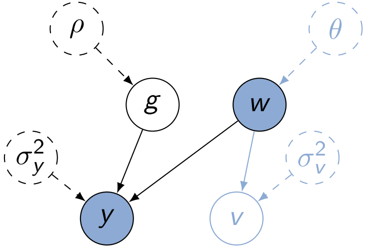




$$\left\{ (\bar{g}^{(0)}, \bar{w}^{(0)}) \right\}$$

Gibbs sampling the UI model





$$g | y, w \sim \mathcal{N}$$

$$\left\{ (\bar{g}^{(0)}, \bar{w}^{(0)}) \quad (\bar{g}^{(1)}, \qquad) \qquad \right\}$$

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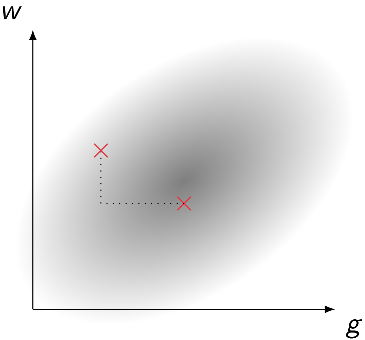

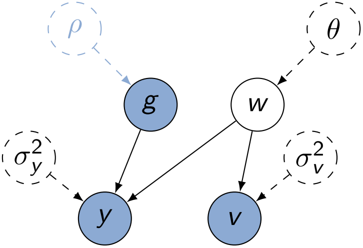

$$w | g, v, y \sim \mathcal{N}$$

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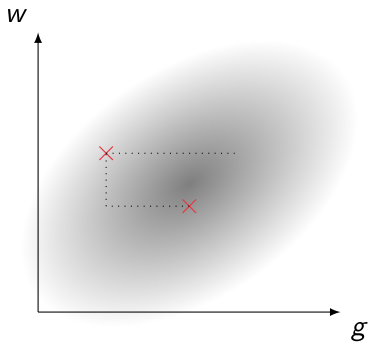

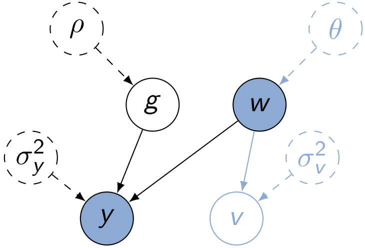

$g|y, w \sim \mathcal{N}$

$$\left\{ (\bar{g}^{(0)}, \bar{w}^{(0)}) \quad (\bar{g}^{(1)}, \bar{w}^{(1)}) \quad (\bar{g}^{(2)}, \qquad) \right\}$$

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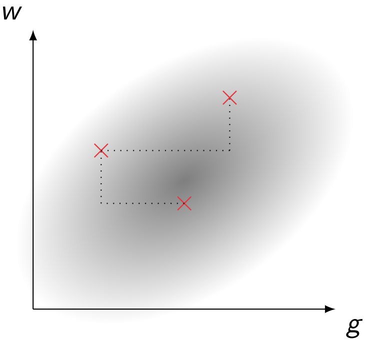

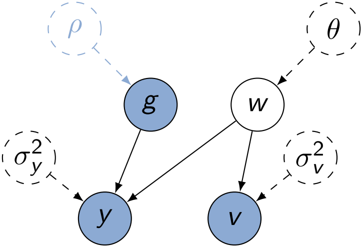

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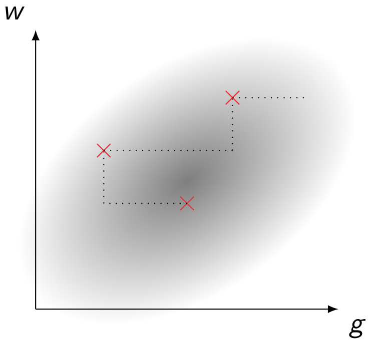

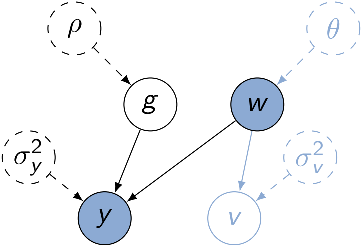

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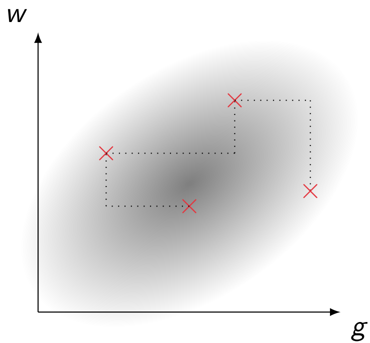

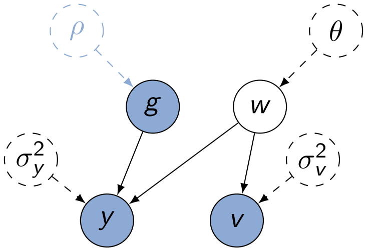

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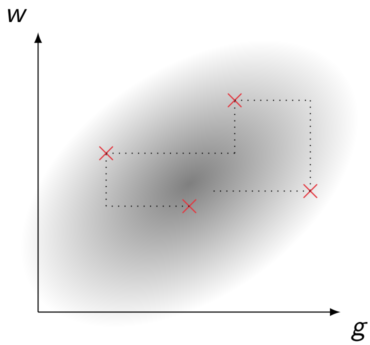

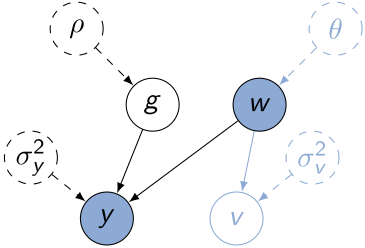

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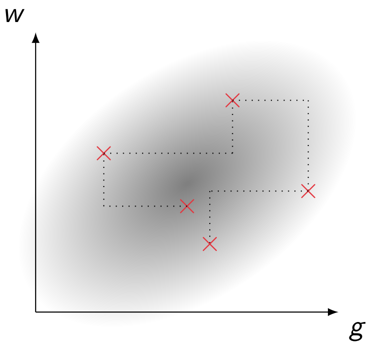

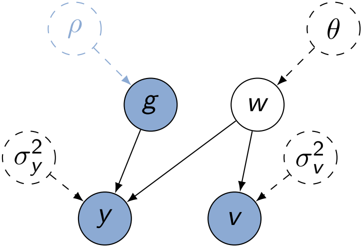

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Maximizing the marginal likelihood

- We want to compute the *marginal likelihood estimate*

$$\hat{\tau} = \arg\max_{\tau} p(y, v; \tau)$$

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A maximum likelihood problem with missing data!

EM for marginal likelihood estimation

- Maximum likelihood problem with missing data

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- We can use the EM-method!

$$\begin{aligned} \text{E-step} \quad & Q(\tau, \tau^{(k)}) = \mathbf{E}\left\{\log p(y, v, g, w; \tau)\right\} \\ & \qquad \text{w.r.t. } p(g, w | y, v; \hat{\tau}^{(k)}) \\ \text{M-step} \quad & \tau^{(k+1)} = \arg\max Q(\tau, \tau^{(k)}) \end{aligned}$$

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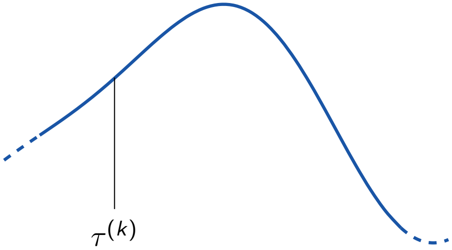

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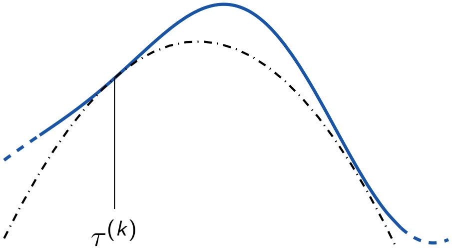

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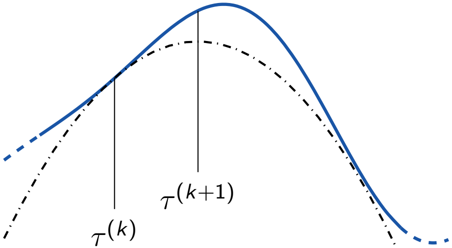

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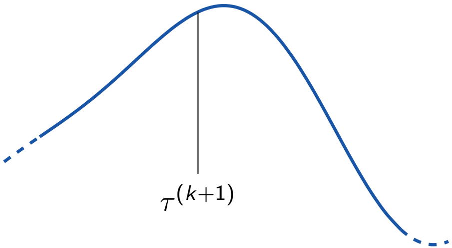

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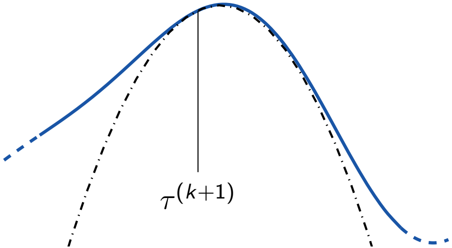

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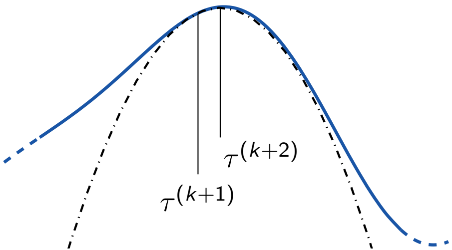
M-step $\quad \tau^{(k+1)} = \arg\max Q(\tau, \tau^{(k)})$



EM for marginal likelihood estimation

- Maximum likelihood problem with missing data

$$\hat{\tau} = \arg\max_{\tau} p(y, v; \tau) = \arg\max_{\tau} \int p(y, v, g, w; \tau)\, \mathrm{d}g\, \mathrm{d}w$$

- We can use the EM-method!

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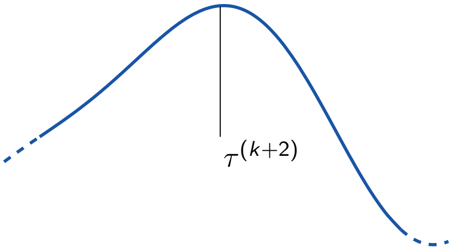

Monte Carlo Expectation Maximization

- The E-step is difficult to evaluate

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Monte Carlo Expectation Maximization

MCEM for hyperparameter estimation

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Monte Carlo inference in UI models

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- Posterior means

$$\{\bar{g}^{(j)}, \bar{w}^{(j)}\}_{j=1}^{M} = \textsc{GibbsSampler}(\hat{\tau})$$

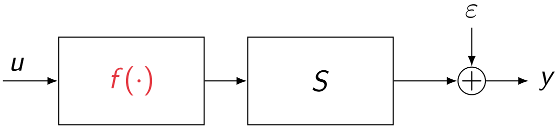
$$\mathbf{E}\{g|y, v\} \approx \frac{1}{M}\sum_{j=1}^{M} \bar{g}^{(j)}$$

$$\mathbf{E}\{w|y, v\} \approx \frac{1}{M}\sum_{j=1}^{M} \bar{w}^{(j)}$$

Example: Hammerstein models

Example: Hammerstein models

- Nonparametric Hammerstein model



Example: Hammerstein models

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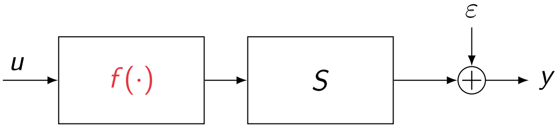

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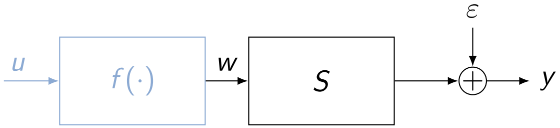
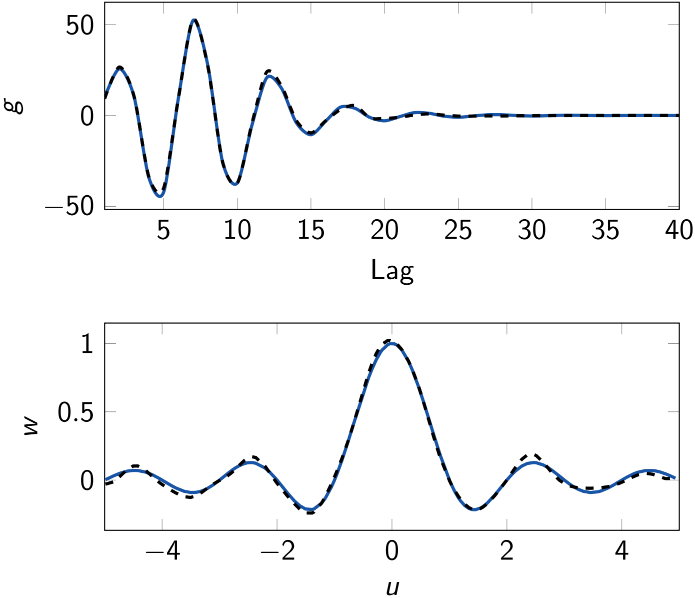


- uncertain-input model with

$$g \sim \mathcal{N}\big(0, K_g(\rho)\big) \quad w \sim \mathcal{N}\big(0, K_w(\theta)\big)$$

$$K_g(\rho)_{i,j} = \underbrace{\rho_1^{\max(i,j)}}_{\text{Stable-spline kernel}} \qquad [K_w(\theta)]_{ij} = \exp\left[-\frac{1}{\theta}(u_i - u_j)^2\right]$$

Example: Hammerstein models



$n_{\mathrm{lti}} = 40,\ N = 400,\ \mathrm{SNR} = 10,$ ▬▬ = true, - - - = estimated

Under the hood

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  1. MC Step: Gibbs sampler
     - Reduce the covariance matrices with SVD (Bad conditioning!)
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- Run new Gibbs sampler
  1. Reduce the covariance matrices with SVD (Bad conditioning!)
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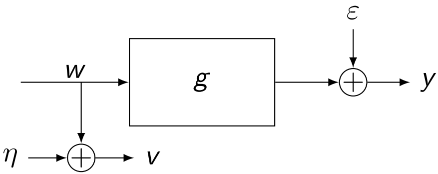
Special classes of systems



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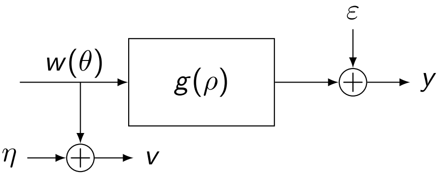

- PIPS: parametric-input parametric-system models

$$K_g(\rho) = 0 \qquad K_w(\theta) = 0$$

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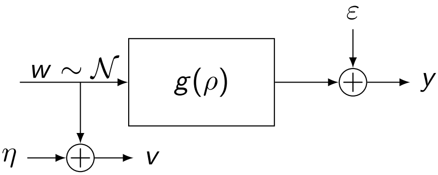

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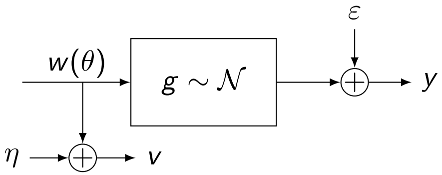

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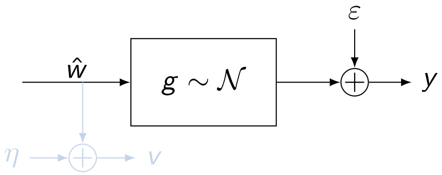

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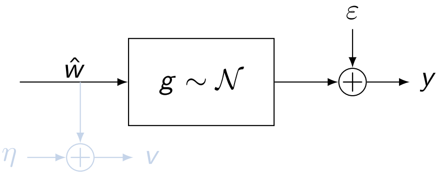
$$w = \hat{w}$$

Special classes of systems



- PIPS: parametric-input parametric-system models

$$K_g(\rho) = 0 \qquad K_w(\theta) = 0 \leftarrow$$ ML and PEM!

- GIPS: Gaussian-input parametric-system models

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- PIGS: parametric-input Gaussian-system models

$$K_w(\theta) = 0 \leftarrow$$ Bayesian FIR models!

- EIGS: Estimated-input Gaussian-system models

$$w = \hat{w}$$

Example: Semi-blind models

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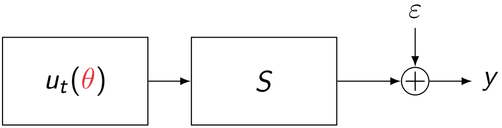
- Semi-blind model



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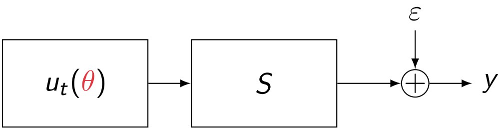
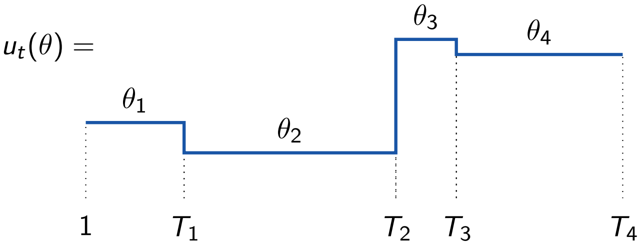


- Piecewise constant input



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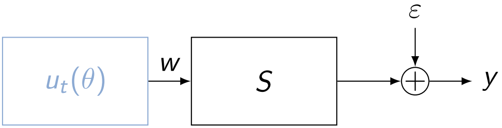

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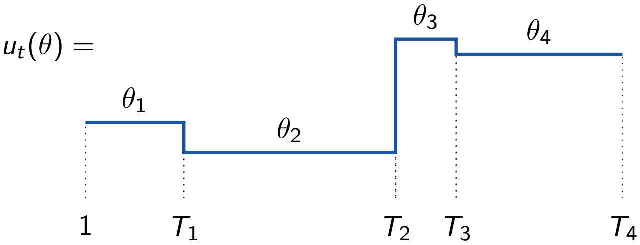
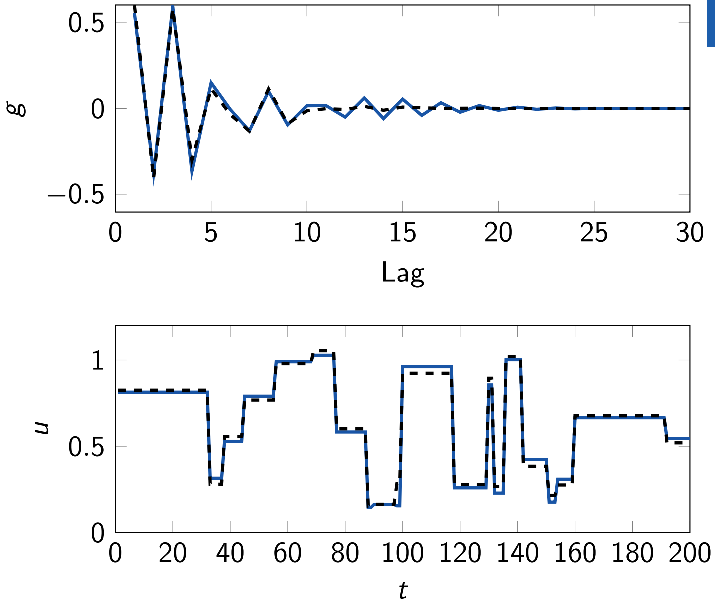


- PIGS uncertain-input model with

$$w \sim \mathcal{N}(H\theta, 0) \qquad g \sim \mathcal{N}(0, K(\rho))$$

Example: Semi-blind models

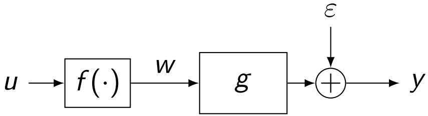


$n_{\mathrm{lti}} = 20,\ N = 200,\ p = 20,\ \mathrm{SNR} = 10,$ —— = true, - - - = estimated

Other examples

- Errors-in-variables
- Cascaded models
- Estimation of initial conditions
- Systems with missing data

Another Hammerstein example



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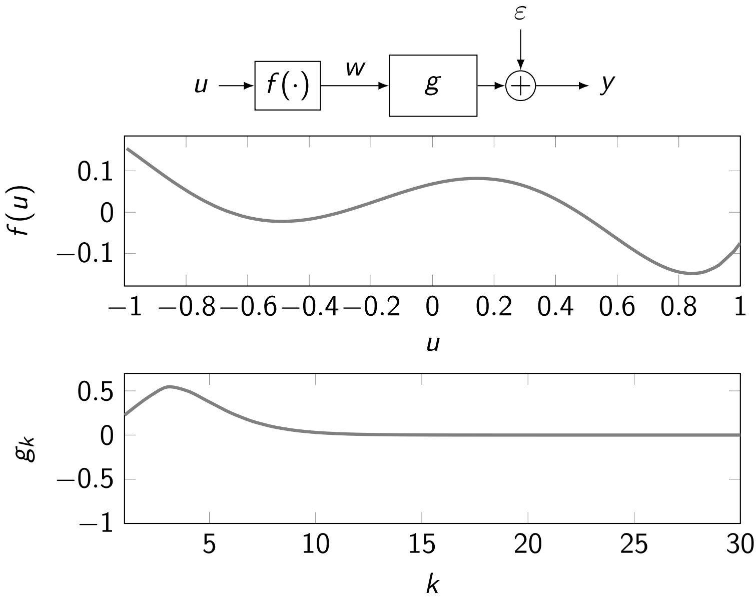

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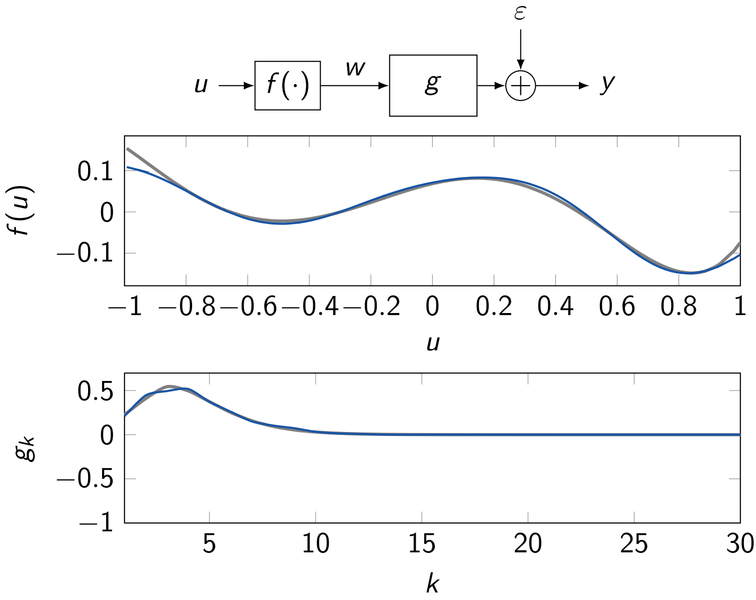

$\varepsilon \sim \mathcal{N}(0, \sigma^2)$

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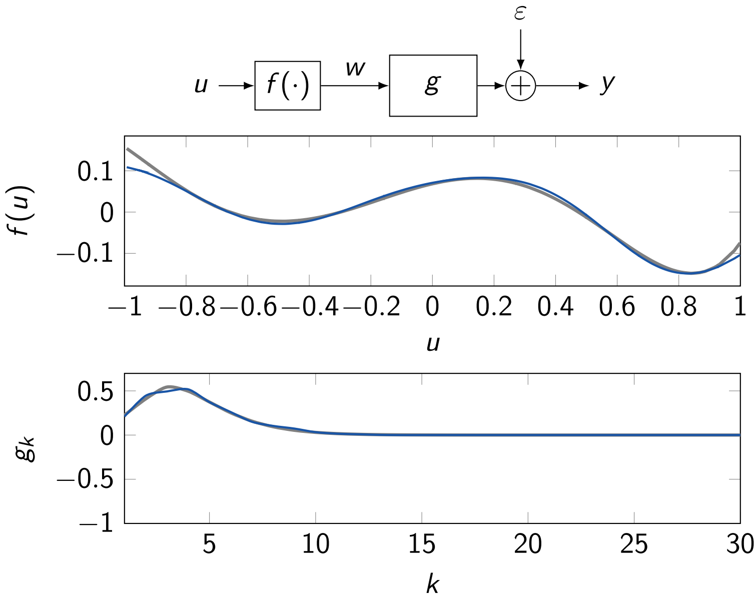

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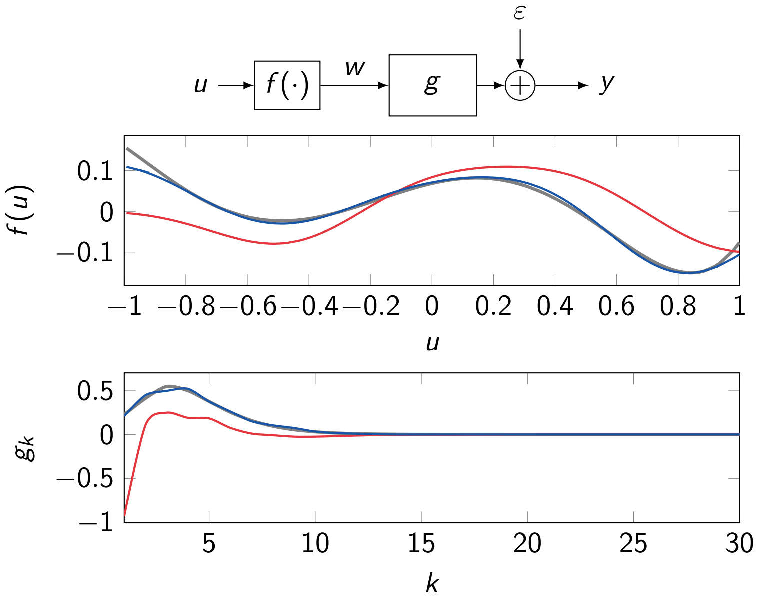

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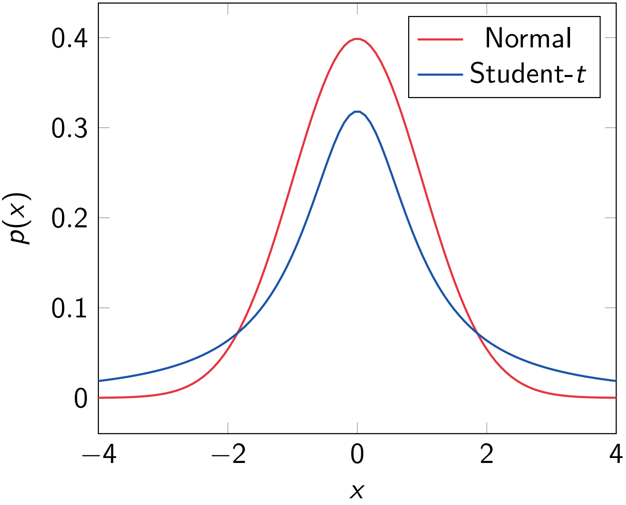
We need a noise model with *heavy tails*



Compounded Gaussian noise model

- Student-t model for the noise

$$\varepsilon_t \sim \mathrm{St}(\nu, \eta)$$

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$$\lambda_t \sim \mathrm{Ga}(\alpha, \beta) \longleftarrow \text{ Prior density!}$$

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$$\begin{aligned} \lambda_t &\sim \mathrm{Ga}(\alpha, \beta) \longleftarrow \text{ Prior density!} \\ \varepsilon_t | \lambda_t &\sim \mathcal{N}\left(0, \lambda_t^{-1}\right) \end{aligned}$$

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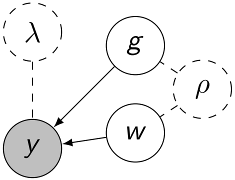

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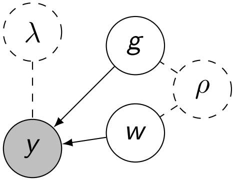

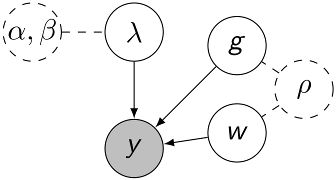

Model estimation

- Priors

$$g \sim \mathcal{N}\big(0,\, K_g(\rho)\big), \quad w \sim \mathcal{N}\big(0,\, K_w(\rho)\big), \quad \lambda_t \sim \mathrm{Ga}(\alpha, \beta)$$

$$t = 1, \ldots, N$$

- Data model

$$y|g, w, \lambda_1, \ldots, \lambda_N \sim \mathcal{N}\big(Wg,\, \mathrm{Diag}\{\lambda_t^{-1}\}\big)$$

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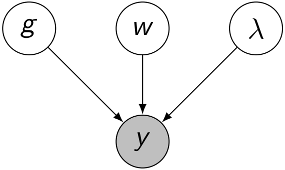
- Same type of model as before only more hyperparameters ($\{\lambda_t\}_{t=1}^N$ instead of σ_y^2)
- Same tools as before can be used (MCEM, Gibbs sampling)

Gibbs sampling



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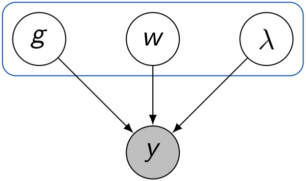

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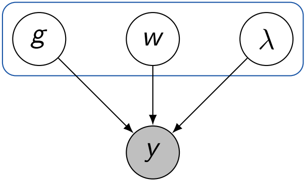
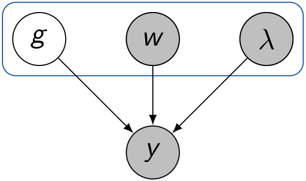


In sequence

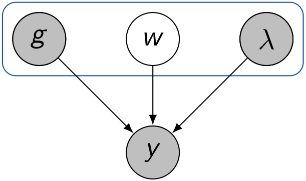
Gibbs sampling



In sequence

1. sample $\bar{g}^{(i)} | \bar{w}^{(i-1)}, \bar{\lambda}^{(i-1)}, y$

Gibbs sampling



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1. sample $\bar{g}^{(i)}|\bar{w}^{(i-1)}, \bar{\lambda}^{(i-1)}, y$
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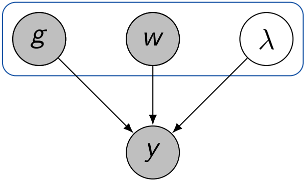

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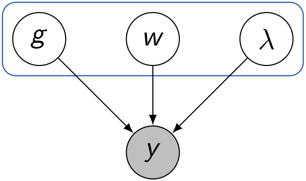
Gibbs sampling



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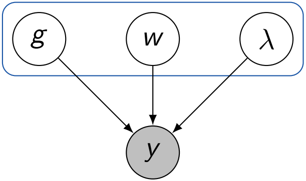
3. sample $\bar{\lambda}^{(i)}|\bar{g}^{(i)}, \bar{w}^{(i)} y$

Result

A Markov chain with $(g, w, \lambda|y)$ as its stationary distribution

Gibbs sampling



In sequence

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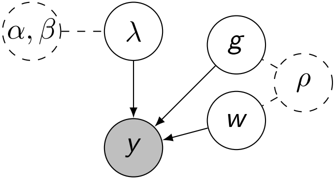
Result

A Markov chain with $(g, w, \lambda|y)$ as its stationary distribution

Known expressions for

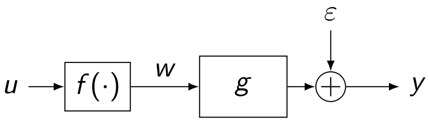
$$g|y, w, \lambda \sim \mathcal{N}(m_g, P_g)$$
$$w|y, g, \lambda \sim \mathcal{N}(m_w, P_w)$$
$$\lambda_t|y, g, w \sim \mathrm{Ga}(\alpha_t, \beta_t)$$

Approximate inference algorithm for Robust UI models




```
procedure ESTIMATE-ROBUST(data)
    Initialize ρ̂, α̂, β̂
    while not converged do                                    ▷ EM
        Approximate Q(ρ, α, β)                                ▷ Gibbs sampling
        ρ̂, α̂, β̂ ← arg max_{ρ,α,β} Q(ρ, α, β)                ▷ Scalar optimization
    end while
    ĝ, ŵ, λ̂ ← E {[} g, w, λ|y; ρ̂, α̂, β]                    ▷ Gibbs sampling
    return ĝ, ŵ, λ̂
end procedure
```

Simulation study: Hammerstein systems



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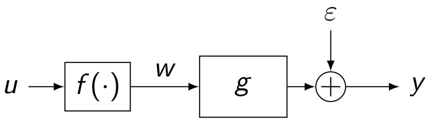

- Polynomial nonlinearity of order $p \in 5, \ldots, 10$

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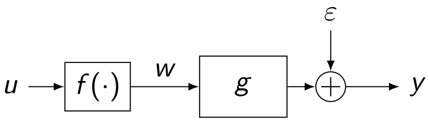

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- Transfer function of order $m \in 3, 4, 5$

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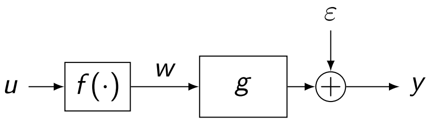

- Polynomial nonlinearity of order $p \in 5, \ldots, 10$
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- $N = 300$ samples of output, uniform white input in $-1, 1$

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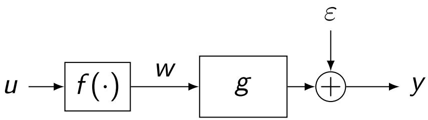

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Two methods

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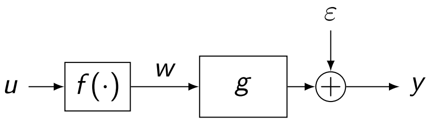

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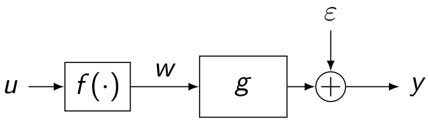
Two methods

H-Gaussian with Gaussian noise model

Simulation study: Hammerstein systems



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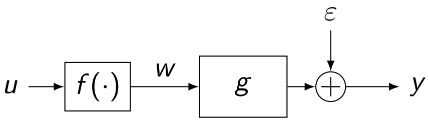

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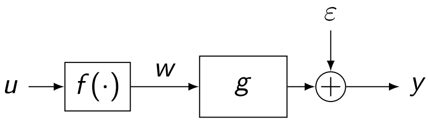

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- Varying number of outliers (with fixed variance $10\sigma^2$)

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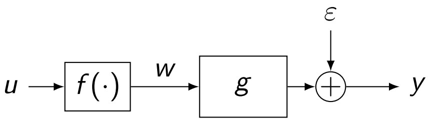

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Two experiments

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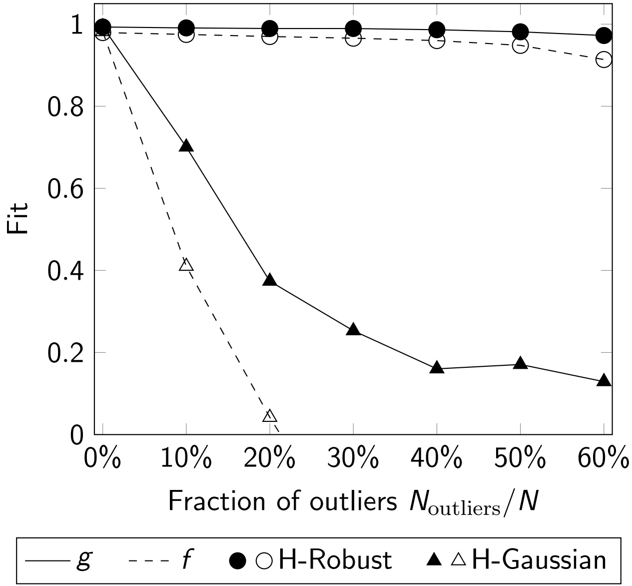
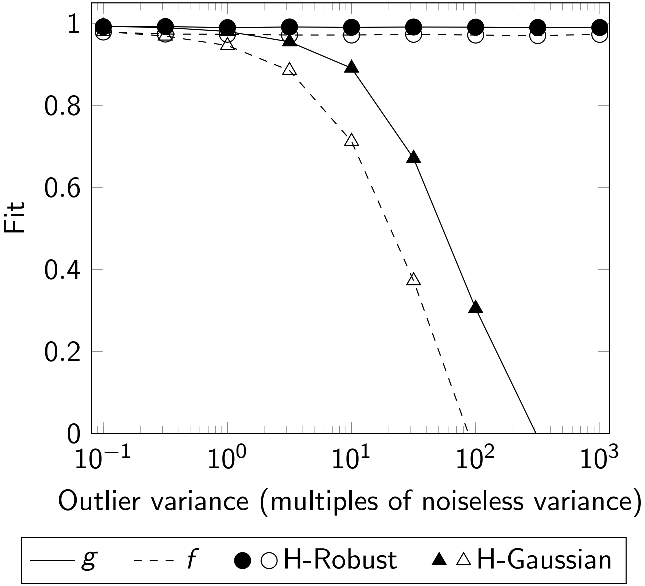
- Varying outlier variance (with fixed fraction 15%)

Outlier fraction results



Outlier variance results



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- Proposed a robust extension with Student-t likelihood